DISSERTATIO ACADEMICA,

DE

FIGURA TELLURIS OPE PENDULORUM DETERMINANDA;

CUJUS

PART. V,

CONS. AMPL. FAC. PHIL. AB.

PRÆSIDE

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IN AUDIT. JURIDICO D. XXVII MAJI MDCCCXV.

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ABOÆ, TYPIS FRENCKELLIANIS.

Antequam ratiociniorum & formularum, quæ in præcedentibus afferuntur, inſtituitur applicatio, plurium noviſſimis maxime his annis factarum obſervationum fiat recenſio, quo plenior earum habeatur collectio & certior inde deducatur concluſio. Maxima earum pars Hiſpaniæ Navigatori nomine *Ciscar* debetur, qui in itinere circum terram invariato utebatur pendulo ligneo, globum inferne ferente cupreum (*). Adeo parva vulgo putatur ligni a calore dilatatio, ut negligi omnino queat, quare pro longitudine hujus penduli nulla opus erit correctione. Neque variationes denſitatis aëris, ſi datæ quoque eſſent, aliquam requirere reductionem oſtendit Cel. *Oltmanns*, cum partem $\frac{1}{310}$ altitudinis Barometricæ numquam ſuperaverit Mercurii variatio. In reductione igitur, parva certe etiam illa, ad ſpatium aëre vacuum & temperaturam congelationis aquæ, ſolius caloris aëris in obſervando habenda eſt ratio, qui,

(*) Vide recenſionem Cel. *Oltmanns* in *v. Zachs Monatl. Corresp.* 1812, p. 468 &c.

cum ab obſervatore non ſit adnotatus, e cognitis legibus, ſecundum quas ab æquatore verſus polos telluris decreſcit, eſt deducendus. Facta ſic longitudine penduli ſimplicis æquatorialis in vacuo $= p$, calore ſub æquatore medio $= m$, & numero oſcillationum penduli invariati $= N_{(m)}$, nec non pondere cupri ſpecifico $= 7,8$, poſitisque, pro alio quovis loco telluris, longitudine penduli ſimplicis ad vacuum reducta $= \pi$, calore ibi veriſimili $= \mu$, atque numero oſcillationum $= N_{\mu}$), erit e præcedentibus $\pi = p \left(\frac{N(\mu)}{N(m)}\right)^2 \times \left(1 + \frac{0,0000507}{7,8+0,02925.\mu}\right) : \left(1 + \frac{0,0000507}{7,8+0,02925.m}\right)$.

Quo loco æquatoris obſervaverit *Ciscar* non innoteſcit; cum valore penduli illius tamen determinationes comparat *Oltmanns*, quem nempe neceſſariis adhibitis correctionibus in regno Peruviano invenit *Bouguer* eſſe $p = 439,21$ lin. Pariſ. . Etiamſi hunc valorem in præcedentibus novis adhuc reductionibus mutandum eſſe judicatum ſit, ex iis tamen rationibus, quæ in ſequentibus proponentur, apparebit veriſimiliorem eſſe eum, qualis poſt reductionem *Bougueri* habetur. Mediam deinde aëris temperaturam in Zona torrida regionis Americanæ, per totum fere annum eandem, obſervavit *Humboldt* (*) ad ſuperficiem maris eſſe $= 27°$ Celſ.,

a

(*) *Voyage de Humboldt & de Bonpland, IVe Partie, I Vol. I Livraiſ. p. 112.*

a qua non multum differt valor a *Kirwan* propoſitus (*), nempe 29° Celſ. His igitur adhibitis valoribus, factoque, ut obſervavit *Ciscar*, $N_{(m)} = 3607$ pro tempore 24 horarum, habebitur

$$\pi = \frac{439,21}{13010524} \cdot \left(N(\mu)\right)^2 \cdot \left(1 + \frac{0,0000507}{7,8 + 0,02925 \cdot \mu}\right), \text{ ſeu}$$

$$\text{etiam } \pi = 0,000033758 \left(N(\mu)\right)^2 \cdot \left(1 + \frac{0,0000507}{7,8 + 0,02925 \cdot \mu}\right).$$

Ipſæ autem obſervationes & inde deducti valores pendulorum heic ſunt:

	Geographica		Calor	Numerus	Longit.
	Latitudo loci	Longitudo a Pariſiis	veriſimilis	oſcillat. in $24^{h.}$	pend. ſimpl. in vacuo
Mulgrave (†	67.°30.'0N.	167°.0'.0"W.	†10°C.	3614,85	441,122
Nootka - -	49. 35. 0	129. 2. 30W.	15	3612,21	440,479
Monterey -	36. 36. 0	124. 3. 0 W.	20	3610,75	440,123
Gades - - -	36. 32. 0	8. 37. 30W.	22	3610,24	439,999
Macao - -	22 12. 44	111. 15. 0O	24	3608,58	439,594
Acapulco -	16. 50. 29	101. 56. 0W	25	3607,83	439,412
Manilla - -	14 36. 8	118. 32. 0O.	26	3607,06	439,224
Umatog - -	13. 18. 0		26	3607,07	439,226

Sam-

(*) In libro ſuo: *Angabe der Temperatur von den verſchiedenen Länder und Städte. Aus d. Engl. Berlin 1788, p. 36.*

(†) Hujus loci latitud. adnotavit Cel. *Oltmanns* eſſe 59° 33'; alium vero hoc nomine inſignitum locum, quam cujus mentionem hic facimus, in vulgaribus Mapp. Geogr. non potuimus reperire.

Sambuangan -	6. 55. 0	119. 0. 0 O.	27	3607,25	439,268
Æquator - - -	0. 0. 0	82. 0. 0 W	27	3607,02	439,210
Lima - - - -	12. 5. 0 S.	79. 9. 30 W	6	3607,39	439,274
Babao ſ. Vavao	18. 39. 0	177. 0. 0. W.	24	3608,12	439,482
Portus Jackſon	33. 51. 0	148 59. 30 O	2	3610,20	439,989
Monte Video -	34. 55. 0	58. 25 0 W	21	3610,38	440,033
Conception - -	36. 42. 0	75. 0. 0 W	20	3610,29	440,011
St. Helena - -	44. 30. 0	67. 40. 0 W	18	3612,37	440,518
Puerto Egmont	51. 21. 0	61. 0. 0 W	15	3612,75	440,611

His addi posſunt ſequentes obſervationes:

Gryphiswaldia	54°. 4'. 35" N.	11°. 14'. 30" O.	440,830
Lugdunum	52. 9. 30	2. 8. 30 O.	440,710
Schweidnitz	50. 50. 39	14. 16. 30 O.	440,635
Formentera	38. 39. 56	1. 8. 0 W.	440,1545
Melita	35. 54. 0	12. 9. 0 O.	440,22

Accuratiore inſtituto examine, quædam de obſervationibus allatis animadverſiones adponendæ videntur. Sicut fortisſimam adesſe infra apparebit rationem, cur valor penduli æquatorialis Americani, qualem eum *Bouguer*, inſtituta ad vacuum correctione, determinavit, ita verisimiliores ejus ſunt retinendi valores pro Portobello = 439, 30 & Parva Goava = 439,47. In obſervationibus autem a *La Caille* inſtitutis error aliquis latere videtur. Numeravit penduli invariabilis oſcillationes fuisſe

Pariſiis in calore + 12°, 5 R. - - -	86453
in Promont. bonæ ſpei - 13 - - - -	86406,79
Portu Ludovici - - 20,5 - - - -	86367
Petropoli - - - - 12,5 - - - -	86508,75.

Si

Si pendulum Parisinum uti terminus comparationis adhibetur, supra allatæ oriuntur pro Promontorio bonæ spei & Portu Ludovici longitudines nimis magnæ. Instituta vero comparatione cum pendulo Petropolitano, habetur pendulum

Promont. bonæ spei - - -	439,976
Port. Ludovici - - - -	439,561.

Et quum longitudo penduli pro Rio Janeiro a pendulo Promontorii bonæ spei sit determinata, in eadem ratione illa est minuenda, quo facto erit in Rio Janeiro pend. = 439,95. Quod vero pendulum in insula Manillæ a duobus observatoribus diversis temporibus determinatum attinet, probabilior esse videtur ejus valor medius inde desumtus = 439,338. His igitur perpensis, omissisque valoribus penduli Gothaici & Genevensis, quorum ille a *Zach* nimis magnus est determinatus, a *La Place* autem justo minor assumtus videtur, & hic ne reductione quidem ad libellam maris instituta ad magnitudinem loco convenientem evehi potest; reliquos legem continuitatis non male sequi perspicuum est.

Ut autem verisimum est, ex his datis ope æquationum supra allatarum determinari posse valorem penduli polaris, ita rem examinanti mox patebit, faciliori calculo idem ope sequentium æquationum quam proxime, & majori quidem certitudine, quam qua pol-

pollent ipſæ obſervationes, obtineri. Nulla nempe de natura denſitatis terræ aſſumta hypotheſi, modo illud ponatur, meridianos omnes terreſtres, utpote a figura circulari parum aberrantes, ellipticos eſſe, demonſtratum eſt (*), incrementum longitudinis penduli pro loco dato ſupra longitudinem penduli æquatorialis propotionale eſſe quadrato Sinus latitudinis loci geographicæ. Facta igitur in eodem meridiano longitudine penduli ad Æquatorem $= E$, ad polum terræ $= P$, in latitudine geographica $l = p$, atque in latitudine $\lambda = \pi$, nec non incremento penduli memorato $= x$, eſt $p = E + x\,Sin\,l^2$, & $\pi = E + x\,Sin\,\lambda^2$, adeoque $P = E + x$. Eliminata autem quantitate E, eruitur $P - p = x\,Cof\,l^2$, & $P - \pi = x\,Cof\,\lambda^2$, unde $(P - p)\,Cof\,\lambda^2 = (P - \pi)\,Cof\,l^2$, atque

$$P = \frac{p\,Cof\,\lambda^2 - \pi\,Cof\,l^2}{Cof\,\lambda^2 - Cof\,l^2},$$

qui valor ad computandum commodior redditur duabus hisce æquationibus:

$$Sin\,\beta = \frac{Cof\,l}{Cof\,\lambda},\quad P = p + (p - \pi)\,Tg\,\beta^2.$$

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(*) *Theorie de la figure de la Terre, par Clairaut, Paris 1743 p. 191. Mechanik des Himmels von La Place, Berlin 1802, Th. 2, p. 121.*

Hujus valoris certitudo inde perſpicitur, quod variationes ſimultaneæ quantitatum p, π & P hac definiantur æquatione: $dP = dp + (dp - d\pi)\, Tg\beta^2$, quæ pro caſu quo maximus oritur effectus variationum, quando ſcilicet p exceſſu æque aberrat a vero ac π defectu, præbet

$$dP = \pm (1 + 2\, Tg\beta^2)\, dp;$$

cumque hic, ut ſupra factum eſt, ſumere liceat $dp = 0{,}02$, habebitur

$$dP = \pm 0{,}02\, (1 + 2\, Tg\beta^2).$$

Omnes ex iis obſervationibus, quæ ſupra ſunt exhibitæ, per combinationem valorum penduli duorum locorum, quæ aut ad eundem pertinent meridianum, aut quorum meridiani ultra 5 fere gradus æquatoriales a ſe non diſtant, computati valores penduli polaris hic collecti oculis ſubjiciantur, cum adjecto valore probabilitatis, quæ reciprocam ſequitur rationem admittendi erroris penduli. Quo inde jam colligatur valor penduli polaris, quisque valor ſpecialis cum ſua probabilitate multiplicetur, & ſumma omnium productorum cum ſumma probabilitatum dividatur, ut in ſequentibus apparebit.

	Valor penduli polaris P.	Probabilitas I / dP	P / dP.
Spitsbergen & Lugdunum	441, 443	42, 176	18618
Spitsbergen & Mulgrave	441, 452	34, 458	15212
Spitsbergen & Roma	441, 444	44, 509	19648
Spitsbergen & Melita	441, 439	45, 307	2000
Spitsbergen & Babao	441, 449	46, 488	20522
Kola & Petropolis	441, 717	15, 921	7033
Ponoi & Archangel.	441, 570	7, 996	3531
Pello & Revalia	441, 505	12, 487	5513
Pello & Dorpatum	441, 466	22, 367	9874
Pello & Pernavia	441, 476	21, 772	9612
Pello & Arensburgum	441, 517	21, 974	9702
Pello & Caput bonæ ſpei	441, 514	31, 457	13888
Petropolis & Dorpatum	441, 930	2, 274	1005
Petropolis & Pernavia	441, 8903	2, 280	1008
Petropolis & Revalia	443, 5914	0, 602	267
Upſalia & Mulgrave	441, 428	13, 251	5850
Upſalia & Gryphisvaldia	441, 096	7, 728	3409
Upſalia & Schweidnitz	441, 359	11, 267	4973
Upſalia & Vienna	441, 362	15, 997	7061
Upſalia & Roma	441, 395	18, 730	8267
Upſalia & Melita	441, 316	22, 24	9818
Upſalia & Cap. bonæ ſpei	441, 437	23, 203	10243
Mulgrave & Gryphisvaldia	441, 338	21, 895	9663
Mulgrave & Schweidnitz	441, 405	23, 030	10165
Mulgrave & Vienna	441, 403	25, 200	11123
Mulgrave & Roma	441, 414	29, 094	12842
Mulgrave & Melita	441, 381	31, 754	14016
Mulgrave & Cap. b. ſpei	441, 432	32, 461	14329
Dorpatum & Cap. b. ſpei	441, 541	32, 475	9482

Per-

Pernavia & Cap. bonæ ſpei	441, 547	21, 480	9484
Lugdunum & Pariſii	441, 706	3, 515	1553
Toloſa	441, 652	8, 222	3631
Formentera	441, 60[illegible]	11, 830	5224
Vavao	441, 59[illegible]	20, 462	9036
Arensburgum & Cap. b. ſpei	441, 495	21, 351	9426
Gryphisv. & Schveidnitz	442, 062	3, 667	1621
Vienna	441, 795	6, 333	2798
Roma	441, 683	11, 678	5158
Melita	441, 503	15, 591	6883
Cap. b. ſpei	441, 684	16, 672	7364
Londinum & Pariſii	441, 299	2, 799	1235
Toloſa	441, 482	7, 524	3322
Formentera	441, 479	11, 151	4923
Vavao	441, 518	19, 867	8752
Schveidnitz & Vienna	441, 382	2, 691	1188
Roma	441, 469	8, 151	3599
Cap. b. ſpei	441, 541	13, 330	5886
Melita	441, 278	12, 205	5386
Nootka & Monterey	441, 147	10, 525	4643
Madagaſcar	441, 339	19, 134	8445
Port. Ludov.	441, 316	17, 703	7813
Pariſii & Toloſa - - -	441, 605	4, 765	2104
Formentera	441, 554	8, 460	3735
Vavao	441, 564	17, 451	7705
Vienna & Roma	441, 519	5, 508	2432
Melita	441, 241	9, 738	4253
Cap. b. ſpei	441, 59[illegible]	10, 792	4766
Toloſa & Formentera	441, 477	3, 754	1657
Vavao	441, 542	13, 134	5799
Monterey & Port. Ludov.	441, 654	7, 756	3425
Melita & Cap. b. ſpei	445, 157	1, 206	537
Megaſaki & Manilla	442, 257	6, 960	3078

Formentera & Vavao	441, 578	9, 557	4210
Roma & Melita	440, 799	4, 221	1861
Megaſaki & Sambuangan	442, 045	8, 209	3629
Puerto Egmont	441, 296	14, 463	6382
Macao & Guarico	442, 072	0, 816	361
Parva Goava	442, 080	1, 21	538
St. Helena	441, 867	12, 753	5635
Conception	441, 262	7, 142	3152
Puerto Egmont	441, 460	18, 723	8266
Guarico & Jamaica	442, 699	0, 530	235
Lima	442, 489	1, 920	850
Conception	441, 333	7 939	3504
Parva Goava & Lima	442, 760	6, 589	2917
Conception	441, 360	8, 330	3677
Jamaica & Porto bello	441, 353	5, 522	2437
Æquator	441, 655	3 989	1762
Lima	442, 411	0, 720	319
Conception	441, 405	8, 455	3732
Acapulco & Pon ichery	442 30[illegible]	1, 100	487
Manilla & Sambuangan	440, 674	1, 276	562
Monte Video	441, 799	8, 207	3626
St. Helena	441, 922	14, 798	6540
Puerto Egmont	441, 520	20, 594	9093
Porto bello & Æquator	442, 480	1, 376	609
Sambuangan & Monte Video	441, 676	9, 444	4171
Puerto Egmont	441, 491	21, 643	9555
Para & Rio Janeiro	443, 897	4, 080	1811
Lima & Conception	441, 519	9, 799	4326
Æquator & Lima	440, 671	1, 120	494
Monte Video & Puerto Egm.	441, 406	13, 285	5864
	441, 4933	1232, 521	544150